TALAT Lecture 2301

Design of Members

Axial force and bending moment

Example 9.3 : Beam-column with eccentric load

6 pages

Advanced Level

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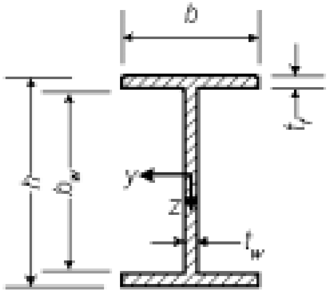
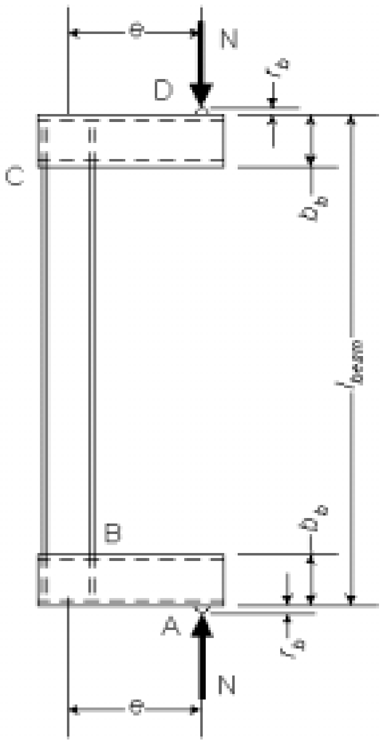
Example 9.3. Beam-column with eccentric load

The beam-column is subject to an eccentric axial force with same eccentricity at both ends. The beam is prevente[d] from warping at the ends by rigid rectangular hollow beams. These beams are simply supported at the load points and D. The structure is a test beam.





Dimensions and material properties

Section height:	$h := 100.5 \cdot mm$
Flange depth:	$b := 50.2 \cdot mm$
Web thickness:	$t_w := 5.07 \cdot mm$
Flange thickness:	$t_f := 5.06 \cdot mm$
Overall length:	$l_{beam} := 800 \cdot mm$
Cross beams:	$b_b := 140 \cdot mm$
Support	$r_b := 30 \cdot mm$
Eccentricity	$exc := 300 \cdot mm$

Alloy: **EN AW-6082 T6** $heat\_treated := 1$ (if heat-treated then 1 else 0)

Note: Values from tests ! $f_{0.2} := 300 \cdot MPa$

[1] (5.4), (5.5) $f_o := f_{0.2}$ $E := 70000 \cdot MPa$ $G := 27000 \cdot MPa$
[1] (5.6)

Partial safety factors: $\gamma_{M1} \equiv 1.10$ $\gamma_{M2} \equiv 1.25$

Inner radius: $r := 0 \cdot mm$

Web height: $b_w := h - 2 \cdot t_f - 2 \cdot r$ $b_w = 90 \cdot mm$

Moment and force

Axial force (compression) $N_{Ed} := 24.8 \cdot kN$

Bending moment at the ends $M_{y.Ed} := N_{Ed} \cdot exc$ $M_{y.Ed} = 7.44 \cdot kNm$

S.I. units: $kN \equiv 1000 \cdot newton$ $kNm \equiv kN \cdot m$ $MPa \equiv 1000000 \cdot Pa$

Cross section class

Axial force

Flange | Web

[1] 5.4.3 and
[1] 5.4.4

$\varepsilon := \frac{250 \cdot MPa}{f_o}$

$\beta_f := \frac{b - t_w}{2 \cdot t_f}$ $\beta_f = 4.459$ $\beta_w := \frac{b_w}{t_w}$ $\beta_w = 17.8$

Heat treated material

$\beta_{1f} := 3 \cdot \varepsilon$ $\beta_{1f} = 2.5$ $\beta_{1w} := 11 \cdot \varepsilon$ $\beta_{1w} = 9.167$

$\beta_{2f} := 4.5 \cdot \varepsilon$ $\beta_{2f} = 3.75$ $\beta_{2w} := 16 \cdot \varepsilon$ $\beta_{2w} = 13.3$

$\beta_{3f} := 6 \cdot \varepsilon$ $\beta_{3f} = 5$ $\beta_{3w} := 22 \cdot \varepsilon$ $\beta_{3w} = 18.3$

$class_f := if(\beta_f > \beta_{1f}, if(\beta_f > \beta_{2f}, if(\beta_f > \beta_{3f}, 4, 3), 2), 1)$ $class_f = 3$ (flange)

$class_w := if(\beta_w > \beta_{1w}, if(\beta_w > \beta_{2w}, if(\beta_w > \beta_{3w}, 4, 3), 2), 1)$ $class_w = 3$ (web)

$class_c := if(class_f < class_w, class_w, class_f)$ $class_c = 3$ (compression)

Bending

As web class > $class_c$ and flange class the same, $class_y := 3$ for *y-y*-axis bending and

$class_z := 3$ for *z-z*-axis bending

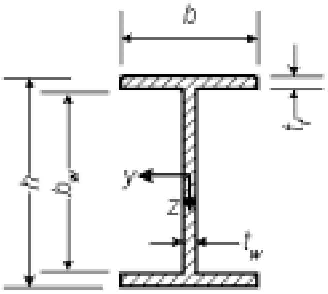
Design resistance, *y-y*-axis bending

[1] 5.6.1 Elastic modulus of the gross cross section W_{el}:

$A_g := 2 \cdot b \cdot t_f + (h - 2 \cdot t_f) \cdot t_w$ $A_g = 966.251 \, mm^2$

$I_g := \frac{1}{12} \cdot \left[b \cdot h^3 - (b - t_w) \cdot (h - 2 \cdot t_f)^3 \right]$ $I_g = 1.47 \cdot 10^6 \, mm^4$

$W_{el} := \frac{I_g \cdot 2}{h}$ $W_{el} = 2.925 \cdot 10^4 \, mm^3$



[1] Tab. 5.3

[1] (5.15) Shape factor α for class 3 cross-section, say $\alpha_y := 1$

[1] (5.14) Design moment of resistance of the cross section $M_{c,Rd}$ $M_{y.Rd} := \frac{f_o \cdot \alpha_y \cdot W_{el}}{\gamma_{M1}}$

$M_{y.Rd} = 8 \, kNm$

Axial force resistance, *y-y* buckling

[1] 5.8.4 Cross section area of gross cross section A_{gr}

$A_{gr} := b \cdot h - (b - t_w) \cdot (h - 2 \cdot t_f)$ $A_{gr} = 966.3 \, mm^2$

Cross section area of class 3 cross section $A_{eff} := A_{gr}$

Effective cross section factor $\eta := \frac{A_{eff}}{A_{gr}}$ $\eta = 1$

Second moment of area of gross cross section:

$I_y := \frac{2}{12} \cdot b \cdot t_f^3 + 2 \cdot b \cdot t_f \cdot \left(\frac{h - t_f}{2} \right)^2 + \frac{1}{12} \cdot (h - 2 \cdot t_f)^3 \cdot t_w$ $I_y = 1.47 \cdot 10^6 \, mm^4$

[1] Table 5.7	Effective buckling length	$l_{yc} := l_{beam} + 2 \cdot r_b$	$l_{yc} = 860 \circ mm$
	Buckling load	$N_{cr} := \dfrac{\pi^2 \cdot E \cdot I_y}{l_{yc}^2}$	$N_{cr} = 1.373 \cdot 10^3 \circ kN$
[1] 5.8.4.1	Slenderness parameter	$\lambda_y := \sqrt{\dfrac{A_{gr} \cdot \eta \cdot f_o}{N_{cr}}}$	$\lambda_y = 0.459$
[1] Table 5.6	$\alpha := if(heat\_treated = 1, 0.2, 0.32)$		$\alpha = 0.2$
	$\lambda_o := if(heat\_treated = 1, 0.1, 0)$		$\lambda_o = 0.1$
	$\phi := 0.5 \cdot \left[1 + \alpha \cdot (\lambda_y - \lambda_o) + \lambda_y^2\right]$		$\phi = 0.642$
	$\chi_y := \dfrac{1}{\phi + \sqrt{\phi^2 - \lambda_y^2}}$		$\chi_y = 0.918$
[1] Table 5.5	Symmetric profile		$k_1 := 1$
[1] Table 5.5	No longitudinal welds		$k_2 := 1$
	Axial force resistance	$N_{y.Rd} := \chi_y \cdot \eta \cdot k_1 \cdot k_2 \cdot \dfrac{f_o}{\gamma_{M1}} \cdot A_{gr}$	$N_{y.Rd} = 241.9 \circ kN$

Axial force resistance, *z-z* axis buckling

	Effective buckling length	$l_{zc} := l_{beam} + r_b$	$l_{zc} = 0.83 \circ m$
	Buckling load $I_z := \dfrac{2 \cdot t_f \cdot b^3}{12}$	$N_{cr} := \dfrac{\pi^2 \cdot E \cdot I_z}{l_{zc}^2}$	$N_{cr} = 107 \circ kN$
[1] 5.8.4.1	Slenderness parameter	$\lambda_z := \sqrt{\dfrac{A_{gr} \cdot \eta \cdot f_o}{N_{cr}}}$	$\lambda_z = 1.646$
[1] 5.8.4.1	$\phi := 0.5 \cdot \left[1 + \alpha \cdot (\lambda_z - \lambda_o) + \lambda_z^2\right]$		$\phi = 2.009$
	$\chi_z := \dfrac{1}{\phi + \sqrt{\phi^2 - \lambda_z^2}}$		$\chi_z = 0.316$
[1] 5.8.4.1	Axial load resistance	$N_{z.Rd} := \chi_z \cdot \eta \cdot k_1 \cdot k_2 \cdot \dfrac{f_o}{\gamma_{M1}} \cdot A_{gr}$	$N_{z.Rd} = 83.352 \circ kN$
	and without column buckling	$N_{Rd} := \eta \cdot \dfrac{f_o}{\gamma_{M1}} \cdot A_{gr}$	$N_{Rd} = 263.5 \circ kN$

Flexural buckling of beam-column

[1] (5.51) $\omega_0 := 1$ $\omega_x := 1$

Exponents in interaction formulae

[1] (5.42c) $\xi_0 := \alpha_y^2$ $\xi_0 := if(\xi_0 < 1, 1, \xi_0)$ $\xi_0 = 1$

[1] 5.9.4.2 $\xi_{yc} := \xi_0 \cdot \chi_y$ $\xi_{yc} := if(\xi_{yc} < 0.8, 0.8, \xi_{yc})$ $\xi_{yc} = 0.918$

Flexural buckling check

$N_{Ed} = 24.8 \circ kN$ $M_{y.Ed} = 7.44 \circ kNm$

[1] 5.4.4 $$U_y := \left(\frac{N_{Ed}}{\chi_y \cdot \omega_x \cdot N_{Rd}}\right)^{\xi_{yc}} + \frac{M_{y.Ed}}{\omega_0 \cdot M_{y.Rd}}$$ $U_y = 1.056$

Lateral-torsional buckling

[1] Annex J Figure J.2

Warping constant: $$I_w := \frac{(h - t_f)^2 \cdot I_z}{4}$$ $I_w = 2.429 \cdot 10^8 \circ mm^6$

Torsional constant: $$I_t := \frac{2 \cdot b \cdot t_f^3 + h \cdot t_w^3}{3}$$ $I_t = 8.702 \cdot 10^3 \circ mm^4$

Lateral buckling length $L := l_{beam} - 2 \cdot b_b$ $W_y := W_{el}$ $W_y = 2.925 \cdot 10^4 \circ mm^3$

Constant bending moment,*warping* prevented at the ends

[1] H.1.2(2) C_1, k and k_w constants $C_1 := 1$ $k := 1$ $k_w := 0.5$

[1] H.1.3(2) $$M_{cr} := \frac{C_1 \cdot \pi^2 \cdot E \cdot I_z}{(k \cdot L)^2} \cdot \sqrt{\left(\frac{k}{k_w}\right)^2 \cdot \frac{I_w}{I_z} + \frac{(k \cdot L)^2 \cdot G \cdot I_t}{\pi^2 \cdot E \cdot I_z}}$$ $L = 520 \circ mm$ $M_{cr} = 27.219 \circ kNm$

[1] 5.6.6.3(3) $$\lambda_{LT} := \sqrt{\frac{\alpha_y \cdot W_y \cdot f_o}{M_{cr}}}$$ $\lambda_{LT} = 0.568$

[1] 5.6.6.3(2) $\alpha_{LT} := if(class_z > 2, 0.2, 0.1)$ $\alpha_{LT} = 0.2$

$\lambda_{0LT} := if(class_z > 2, 0.4, 0.6)$ $\lambda_{0LT} = 0.4$

[1] 5.6.6.3(1) $\phi_{LT} := 0.5 \cdot \left[1 + \alpha_{LT} \cdot (\lambda_{LT} - \lambda_{0LT}) + \lambda_{LT}^2\right]$ $\phi_{LT} = 0.678$

$$\chi_{LT} := \frac{1}{\phi_{LT} + \sqrt{\phi_{LT}^2 - \lambda_{LT}^2}}$$ $\chi_{LT} = 0.954$

Lateral-torsional buckling of beam-column

[1] (5.50) $\omega_x = 1$ $\omega_{xLT} := 1$ $\omega_0 = 1$

[1] 5.9.4.3 Exponents in interaction expression (simply) $\eta_c := 0.8$ $\gamma_c := 1$ $\xi_{zc} := 0.8$

[1] (5.43)

$$U_z := \left[\left(\frac{N_{Ed}}{\chi_z \cdot \omega_x \cdot N_{Rd}}\right)^{\eta_c} + \left(\frac{M_{y.Ed}}{\chi_{LT} \cdot \omega_{xLT} \cdot M_{y.Rd}}\right)^{\gamma_c} + \left(\frac{M_{z.Ed}}{\omega_0 \cdot M_{z.Rd}}\right)^{\xi_{zc}}\right]$$

Utilisation, lateral-torsional buckling $U_z = 1.357$

Utilisation, flexural buckling $U_y = 1.056$

Comment: $U_z = 1.357$ mean that the failure load at the test exceeded the characteristic strength with 35.7