TALAT Lecture 2301

Design of Members

Axial force and bending moment

Example 9.4 : Beam-column with cross weld

7 pages

Advanced Level

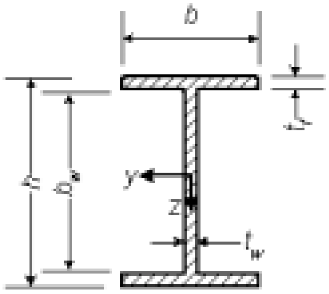
prepared by Torsten Höglund, Royal Institute of Technology, Stockholm

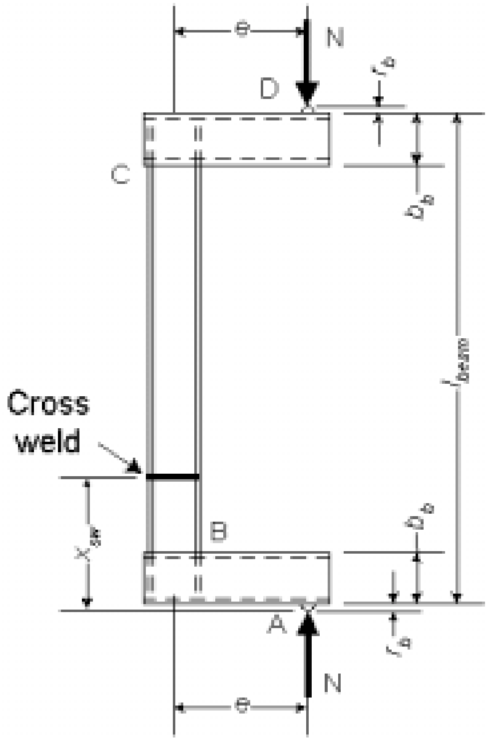
Date of Issue: 1999

Example 9.4. Beam-column with cross weld

The beam-column is subject to an eccentric axial force with same eccentricity at both ends. The beam is prevent
from warping at the ends by rigid rectangular hollow beams. These beams are simply supported at the load point
and D. The structure is a text beam.

There is a welded joint at a section x_{sw} from the support A.





Dimensions and material properties

Section height:	$h := 100 \cdot mm$
Flange depth:	$b := 49.9 \cdot mm$
Web thickness:	$t_w := 4.88 \cdot mm$
Flange thickness:	$t_f := 4.97 \cdot mm$
Overall length:	$l_{beam} := 1201 \cdot mm$
Cross beams:	$b_b := 140 \cdot mm$
Support	$r_b := 30 \cdot mm$
Eccentricity	$exc := 200 \cdot mm$
Location of cross weld	$x_{sw} := 170 \cdot mm$

Alloy: **EN AW-6082 T6** $heat\_treated := 1$ (if heat-treated then 1 else 0)

Note: Values from tests ! $f_{0.2} := 311 \cdot MPa$ $f_{haz} := 223 \cdot MPa$

[1] (5.4), (5.5)
[1] (5.6) $f_o := f_{0.2}$ $f_a := f_{haz}$ $E := 70000 \cdot MPa$ $G := 27000 \cdot MPa$

Partial safety factors: $\gamma_{M1} \equiv 1.10$ $\gamma_{M2} \equiv 1.25$

Inner radius: $r := 0 \cdot mm$

Web height: $b_w := h - 2 \cdot t_f - 2 \cdot r$ $b_w = 90 \cdot mm$

Moment and force

Axial force (compression) $N_{Ed} := 17 \cdot kN$

Bending moment at the ends $M_{y.Ed} := N_{Ed} \cdot exc$ $M_{y.Ed} = 3.4 \circ kNm$

S.I. units: $kN \equiv 1000 \cdot newton$ $kNm \equiv kN \cdot m$ $MPa \equiv 1000000 \cdot Pa$

Classification of the cross section

(Detail calculations omitted)

y-y-axis bending	$class_y := 3$
z-z-axis bending	$class_z := 3$
axial force	$class_c := 3$

Cross weld

[1] 5.5 HAZ softening at a section $x_{sw} = 170 \circ mm$ from the column end

[1] Table 5.2 $\rho_{haz} := 0.65$ *In this example use value from tests* $f_{haz} = 223 \circ MPa$

which means $\rho_{haz} := \dfrac{f_{haz} \cdot \gamma_{M2}}{f_o \cdot \gamma_{M1}}$ $\rho_{haz} = 0.815$ $f_u := \dfrac{f_{haz} \cdot \gamma_{M2}}{\rho_{haz} \cdot \gamma_{M1}}$ $f_u = 311 \circ MPa$

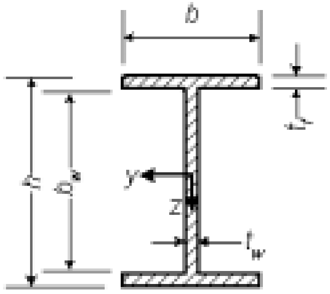
Design resistance, *y-y*-axis bending

[1] 5.6.1 Elastic modulus of the gross cross section W_{el}:

$A_g := 2 \cdot b \cdot t_f + (h - 2 \cdot t_f) \cdot t_w$ $A_g = 935.499 \circ mm^2$

$I_g := \dfrac{1}{12} \cdot \left[b \cdot h^3 - (b - t_w) \cdot (h - 2 \cdot t_f)^3 \right]$ $I_g = 1.418 \cdot 10^6 \circ mm^4$

$W_{el} := \dfrac{I_g \cdot 2}{h}$ $W_{el} = 2.836 \cdot 10^4 \circ mm^3$



[1] Tab. 5.3

[1] (5.15) Shape factor α for class 3 cross-section, say $\alpha_y := 1$

[1] (5.14) Design moment of resistance of the cross section $M_{c,Rd}$ $M_{y.Rd} := \dfrac{f_o \cdot \alpha_y \cdot W_{el}}{\gamma_{M1}}$

$M_{y.Rd} = 8 \circ kNm$

Design resistance, *z-z*-axis bending

Cross section class $class_z = 3$

Gross cross section: $I_z := 2 \cdot \dfrac{t_f \cdot b^3}{12}$ $I_z = 1.029 \cdot 10^5 \circ mm^4$

Section module: $W_z := \dfrac{I_z \cdot 2}{b}$ $W_z = 4.125 \cdot 10^3 \circ mm^3$

Shape factor: $\alpha_z := 1$

Bending resistance: $M_{z.Rd} := \dfrac{f_o \cdot \alpha_z \cdot W_z}{\gamma_{M1}}$ $M_{z.Rd} = 1.166 \circ kNm$

Axial force resistance, *y-y* buckling

[1] 5.8.4 Cross section area of gross cross section A_{gr}

$$A_{gr} := b \cdot h - (b - t_w) \cdot (h - 2 \cdot t_f) \qquad A_{gr} = 935.499 \, mm^2$$

Cross section area of class 3 cross section $A_{eff} := A_{gr}$

Effective cross section factor $\eta := \frac{A_{eff}}{A_{gr}}$ $\quad \eta = 1$

Second moment of area of gross cross section:

$$I_y := \frac{2}{12} \cdot b \cdot t_f^3 + 2 \cdot b \cdot t_f \cdot \left(\frac{h - t_f}{2}\right)^2 + \frac{1}{12} \cdot (h - 2 \cdot t_f)^3 \cdot t_w \qquad I_y = 1.418 \cdot 10^6 \, mm^4$$

[1] Table 5.7 Effective buckling length $l_{yc} := l_{beam} + r_b$ $\quad l_{yc} = 1.231 \text{ m}$

Buckling load $N_{cr} := \frac{\pi^2 \cdot E \cdot I_y}{l_{yc}^2}$ $\quad N_{cr} = 646.436 \, kN$

[1] 5.8.4.1 Slenderness parameter $\lambda_y := \sqrt{\frac{A_{gr} \cdot \eta \cdot f_o}{N_{cr}}}$ $\quad \lambda_y = 0.671$

[1] Table 5.6 $\alpha := if(heat\_treated = 1, 0.2, 0.32)$ $\quad \alpha = 0.2$

$\lambda_o := if(heat\_treated = 1, 0.1, 0)$ $\quad \lambda_o = 0.1$

$\phi := 0.5 \cdot \left[1 + \alpha \cdot (\lambda_y - \lambda_o) + \lambda_y^2\right]$ $\quad \phi = 0.782$

$\chi_y := \frac{1}{\phi + \sqrt{\phi^2 - \lambda_y^2}}$ $\quad \chi_y = 0.844$

[1] Table 5.5 Symmetric profile $k_1 := 1$

[1] Table 5.5 No longitudinal welds $k_2 := 1$

Axial force resistance $N_{y.Rd} := \chi_y \cdot \eta \cdot k_1 \cdot k_2 \cdot \frac{f_o}{\gamma_{M1}} \cdot A_{gr}$ $\quad N_{y.Rd} = 223.4 \, kN$

Axial force resistance, *z-z* axis buckling

	Effective buckling length	$l_{zc} := l_{beam} + r_b$	$l_{zc} = 1.231 \cdot m$
	Buckling load	$N_{cr} := \dfrac{\pi^2 \cdot E \cdot I_z}{l_{zc}^2}$	$N_{cr} = 46.9 \cdot kN$
[1] 5.8.4.1	Slenderness parameter	$\lambda_z := \sqrt{\dfrac{A_{gr} \cdot \eta \cdot f_o}{N_{cr}}}$	$\lambda_z = 2.49$
[1] 5.8.4.1	$\phi := 0.5 \cdot \left[1 + \alpha \cdot \left(\lambda_z - \lambda_o\right) + \lambda_z^2\right]$		$\phi = 3.839$
	$\chi_z := \dfrac{1}{\phi + \sqrt{\phi^2 - \lambda_z^2}}$		$\chi_z = 0.148$
[1] 5.8.4.1	Axial load resistance	$N_{z.Rd} := \chi_z \cdot \eta \cdot k_1 \cdot k_2 \cdot \dfrac{f_o}{\gamma_{M1}} \cdot A_{gr}$	$N_{z.Rd} = 39.118 \cdot kN$
	and without column buckling	$N_{Rd} := \eta \cdot \dfrac{f_o}{\gamma_{M1}} \cdot A_{gr}$	$N_{Rd} = 264.5 \cdot kN$

Flexural buckling of beam-column

[1] 5.8.4.1 Cross weld from column end $x_{sw} = 170 \cdot mm$ $l_{yc} = 1.231\ \mathrm{m}$ $\dfrac{x_{sw}}{l_{yc}} = 0.138$

[1] 5.9.4.5 HAZ reduction factors, see above $\rho_{haz} = 0.815$

[1] (5.51) $\omega_0 := \rho_{haz} \cdot \dfrac{f_u}{\gamma_{M2}} \cdot \dfrac{\gamma_{M1}}{f_o}$ $\omega_0 := if\left(\omega_0 > 1, 1, \omega_0\right)$ $\omega_0 = 0.717$

[1] (5.49) $\omega_x := \dfrac{\omega_0}{\chi_y + \left(1 - \chi_y\right) \cdot \sin\left(\dfrac{\pi \cdot x_{sw}}{l_{yc}}\right)}$ $\omega_x = 0.788$

Exponents in interaction formulae

[1] (5.42c) $\xi_0 := \alpha_y^2$ $\xi_0 := if\left(\xi_0 < 1, 1, \xi_0\right)$ $\xi_0 = 1$

[1] 5.9.4.2 $\xi_{yc} := \xi_0 \cdot \chi_y$ $\xi_{yc} := if\left(\xi_{yc} < 0.8, 0.8, \xi_{yc}\right)$ $\xi_{yc} = 0.844$

Flexural buckling check

$N_{Ed} = 17 \cdot kN$ $M_{y.Ed} = 3.4 \cdot kNm$

[1] 5.4.4 $U_y := \left(\dfrac{N_{Ed}}{\chi_y \cdot \omega_x \cdot N_{Rd}}\right)^{\xi_{yc}} + \dfrac{M_{y.Ed}}{\omega_0 \cdot M_{y.Rd}}$ $U_y = 0.73$

Lateral-torsional buckling

[1] Annex J Figure J.2	Warping constant:	$I_w := \frac{(h - t_f)^2 \cdot I_z}{4}$	$I_w = 2.324 \cdot 10^8 \, mm^6$
	Torsional constant:	$I_t := \frac{2 \cdot b \cdot t_f^3 + h \cdot t_w^3}{3}$	$I_t = 7.958 \cdot 10^3 \, mm^4$
	Lateral buckling length	$L := l_{beam} - 2 \cdot b_b$ $\quad W_y := W_{el}$	$W_y = 2.836 \cdot 10^4 \, mm^3$

Constant bending moment, *warping* prevented at the ends

[1] H.1.2(2) $\quad C_1$, k and k_w constants $\quad C_1 := 1 \quad k := 1 \quad k_w := 0.5$

[1] H.1.3(2)

$$M_{cr} := \frac{C_1 \cdot \pi^2 \cdot E \cdot I_z}{(k \cdot L)^2} \cdot \sqrt{\left(\frac{k}{k_w}\right)^2 \cdot \frac{I_w}{I_z} + \frac{(k \cdot L)^2 \cdot G \cdot I_t}{\pi^2 \cdot E \cdot I_z}} \qquad L = 921 \, mm \qquad M_{cr} = 9.026 \, kNm$$

[1] 5.6.6.3(3)

$$\lambda_{LT} := \sqrt{\frac{\alpha_y \cdot W_y \cdot f_o}{M_{cr}}} \qquad \lambda_{LT} = 0.988$$

[1] 5.6.6.3(2)

$$\alpha_{LT} := if(class_z > 2, 0.2, 0.1) \qquad \alpha_{LT} = 0.2$$

$$\lambda_{0LT} := if(class_z > 2, 0.4, 0.6) \qquad \lambda_{0LT} = 0.4$$

[1] 5.6.6.3(1)

$$\phi_{LT} := 0.5 \cdot \left[1 + \alpha_{LT} \cdot (\lambda_{LT} - \lambda_{0LT}) + \lambda_{LT}^2\right] \qquad \phi_{LT} = 1.047$$

$$\chi_{LT} := \frac{1}{\phi_{LT} + \sqrt{\phi_{LT}^2 - \lambda_{LT}^2}} \qquad \chi_{LT} = 0.718$$

Check three sections

$$i := 1..3 \qquad x_{s_1} := 0 \cdot mm \qquad x_{s_2} := x_{sw} \qquad x_{s_3} := \frac{l_{beam}}{2} + r_b$$

$$l_{zc} := l_{beam} + 2 \cdot r_b \qquad \frac{x_s^T}{l_{zc}} = (0 \quad 0.135 \quad 0.5)$$

HAZ reduction factors $\qquad$ (ω_0 = 1 except at x_{s_2})

[1] (5.51)

$$\omega_{0_i} := \overrightarrow{\left(\rho_{haz} \cdot \frac{f_u}{\gamma_{M2}} \cdot \frac{\gamma_{M1}}{f_o}\right)} \qquad \omega_{0_i} := if(\omega_{0_i} > 1, 1, \omega_{0_i})$$

Weld at section $i = 2$

$$\omega_{0_i} := if(i = 2, \omega_{0_i}, 1) \qquad \omega_0^T = (1 \quad 0.717 \quad 1)$$

[1] (5.49)
or (5.52)

$$\omega_x := \overrightarrow{\frac{\omega_0}{\chi_z + (1-\chi_z)\cdot\sin\left(\frac{\pi\cdot x_s}{l_{zc}}\right)}} \qquad \omega_x^T = (6.76 \quad 1.44 \quad 1)$$

[1] (5.50)
or (5.53)

$$\omega_{xLT} := \overrightarrow{\frac{\omega_0}{\chi_{LT} + (1-\chi_{LT})\cdot\sin\left(\frac{\pi\cdot x_s}{l_{zc}}\right)}} \qquad \omega_{xLT}^T = (1.39 \quad 0.86 \quad 1)$$

[1] (5.42a) $\eta_0 := \alpha_z^2\cdot\alpha_y^2$ $\quad \eta_0 := if(\eta_0<1,1,if(\eta_0>2,2,\eta_0))$ $\quad \eta_0 = 1$

[1] (5.42b) $\gamma_0 := \alpha_z^2$ $\quad \gamma_0 := if(\gamma_0<1,1,if(\gamma_0>2,2,\gamma_0))$ $\quad \gamma_0 = 1$

[1] (5.42c) $\xi_0 := \alpha_y^2$ $\quad \xi_0 := if(\xi_0<1,1,\xi_0)$ $\quad \xi_0 = 1$

[1] 5.9.4.3 $\eta_c := \eta_0\cdot\chi_z$ $\quad \eta_c := if(\eta_c<0.8,0.8,\eta_c)$ $\quad \chi_y = 0.844$ $\quad \eta_c = 0.8$

$\gamma_c := \gamma_0$ $\quad \chi_z = 0.148$ $\quad \gamma_c = 1$

$\xi_{zc} := \xi_0\cdot\chi_z$ $\quad \xi_{zc} := if(\xi_{zc}<0.8,0.8,\xi_{zc})$ $\quad \xi_{zc} = 0.8$

Lateral-torsional buckling of beam-column

$M_{z.Ed} := 0\cdot kNm$

[1] (5.43)

$$U_{LT} := \overrightarrow{\left[\left(\frac{N_{Ed}}{\chi_z\cdot\omega_x\cdot N_{Rd}}\right)^{\eta_c} + \left(\frac{M_{y.Ed}}{\chi_{LT}\cdot\omega_{xLT}\cdot M_{y.Rd}}\right)^{\gamma_c} + \left(\frac{M_{z.Ed}}{\omega_0\cdot M_{z.Rd}}\right)^{\xi_{zc}}\right]} \qquad U_{LT}^T = (0.535 \quad 1.071 \quad 1.104)$$

Max utilisation, lateral-torsional buckling $\quad U_{z.max} := max(U_{LT})$ $\quad U_{z.max} = 1.104$

Max utilisation, flexural buckling $\quad U_y = 0.73$

Comment: $U_{z.max} = 1.104$ mean that the test failure load exceeded the characteristic strength with 10.4%