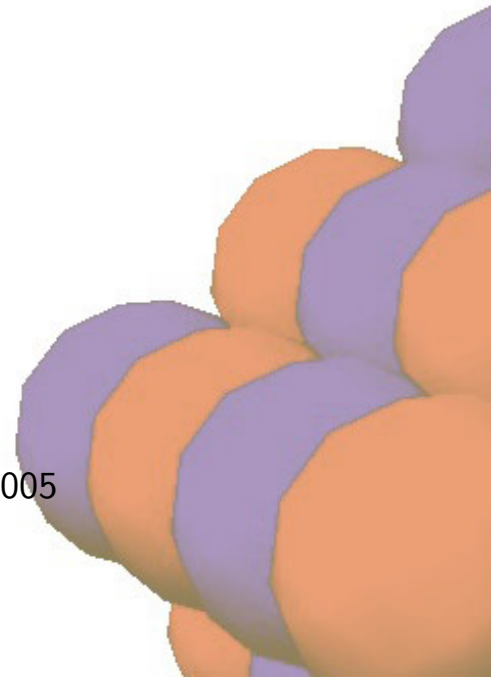


# **AtomicControl: A Crystallography Simulator**

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# Outline

- Motivations and objectives
- Capabilities
- Software model
- Building crystals
- Generating an x-ray diffraction pattern
- Classroom demos

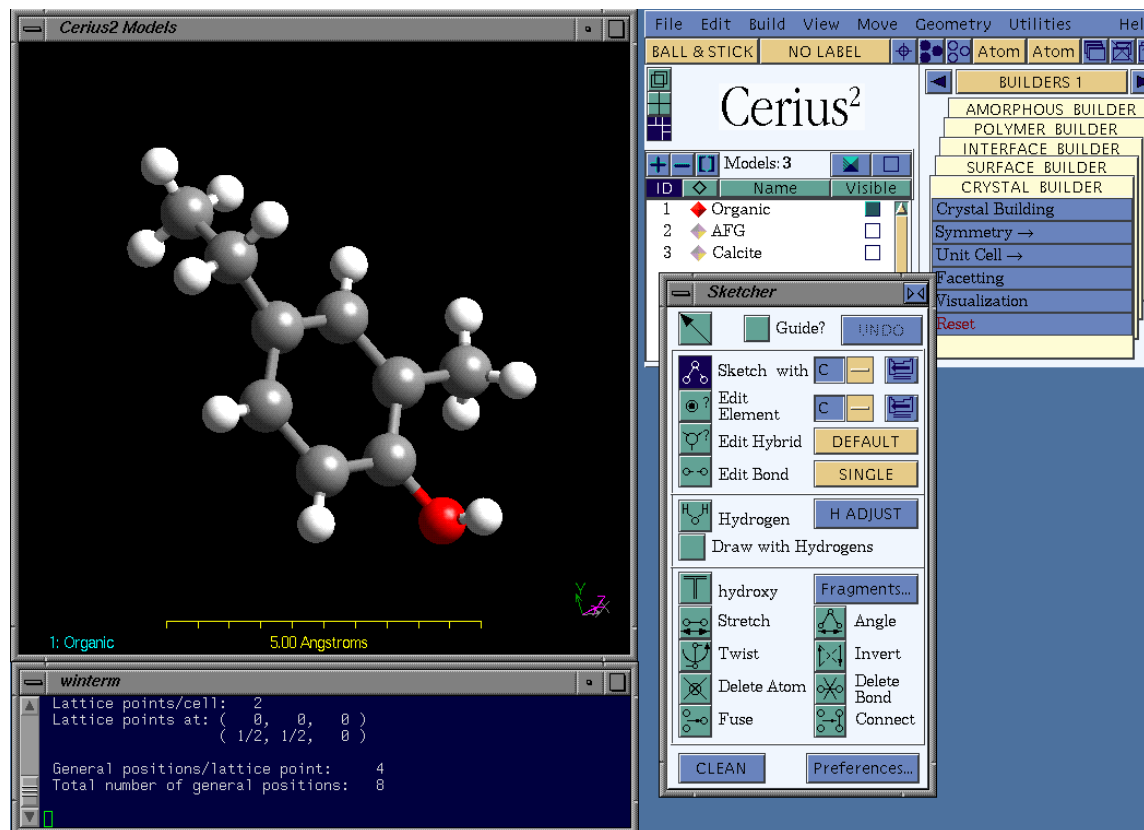
# Motivation

- Visualizing crystal structures is difficult without models
- Instant feedback and interactivity useful for learning
- Classroom use in 3.012 and 3.014

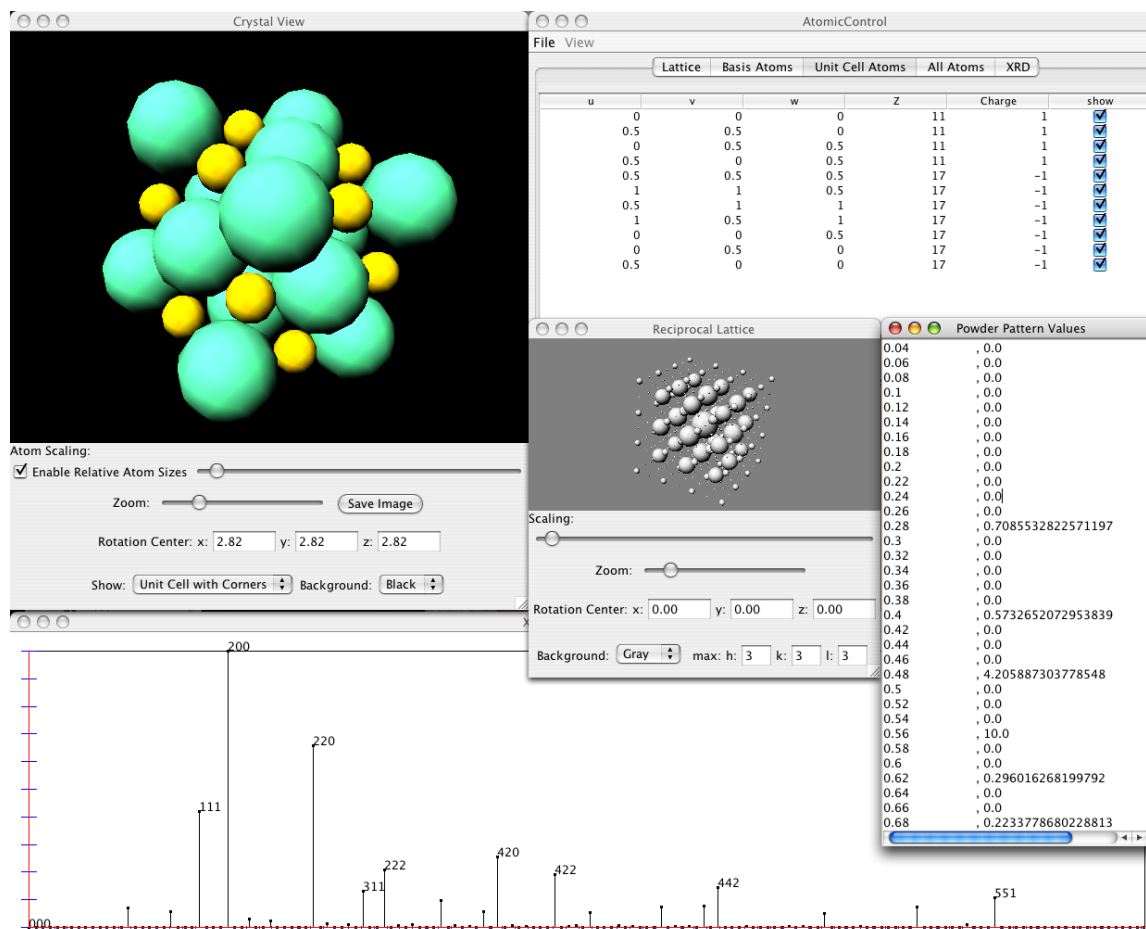
## Objectives of AtomicControl

- Interactively build arbitrary crystal structures
- Visualize crystal structures
- Simulate X-ray diffraction patterns

# Cerius<sup>2</sup>



# AtomicControl



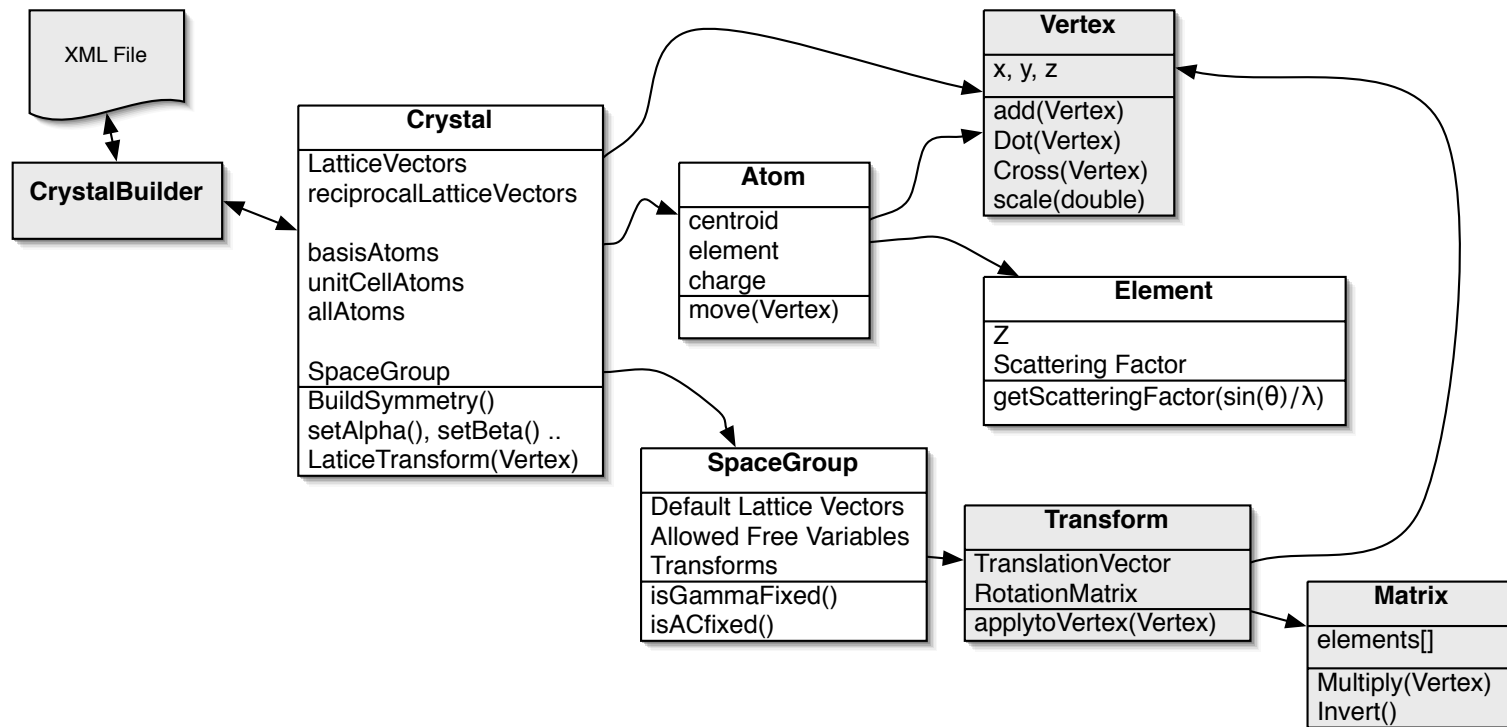
# AtomicControl Capabilities

- Model arbitrary crystal structures
- Save and load previously created structures
- Models allow free rotation of crystal
- Instant visual feedback on modifications
- Relative atom sizes can be represented
- X-ray diffraction patterns generated
- Reciprocal lattice visualization

# Software Model

- Programming Language: Java
- Cross-Platform: Windows, Mac OS X, Linux, Athena
- Modularity
  - Crystal Builder, diffractometer, and graphical interface independant
  - Standardized interfaces for interacting between objects
  - Main module Interface: `Crystal`

# Software Model





# Building Crystals: Space Groups

A space group can be mathematically defined as a set of equipoint transformations of the form:

$$\begin{bmatrix} u' \\ v' \\ w' \end{bmatrix} = \begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix} + \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix}$$

## Positions

Multiplicity,  
Wyckoff letter,  
Site symmetry

## Coordinates

$(0, 0, 0) + (\frac{1}{2}, \frac{1}{2}, \frac{1}{2}) +$

## Reflection conditions

### General:

$$hkl: h + k + l = 2n$$

$$hk0: h, k = 2n$$

$$0kl: k + l = 2n$$

$$hhl: 2h + l = 4n$$

$$00l: l = 4n$$

$$h00: h = 2n$$

$$h\bar{h}0: h = 2n$$

32 *i* 1

- |  |  |
|--|--|
| (1) $x, y, z$  | (2) $\bar{x} + \frac{1}{2}, \bar{y} + \frac{1}{2}, z + \frac{1}{2}$  |
| (3) $\bar{y}, x + \frac{1}{2}, z + \frac{1}{4}$                | (4) $y + \frac{1}{2}, \bar{x}, z + \frac{3}{4}$                      |
| (5) $\bar{x} + \frac{1}{2}, y, \bar{z} + \frac{3}{4}$          | (6) $x, \bar{y} + \frac{1}{2}, \bar{z} + \frac{1}{4}$                |
| (7) $y + \frac{1}{2}, x + \frac{1}{2}, \bar{z} + \frac{1}{2}$  | (8) $\bar{y}, \bar{x}, \bar{z}$                                      |
| (9) $\bar{x}, \bar{y} + \frac{1}{2}, \bar{z} + \frac{1}{4}$    | (10) $x + \frac{1}{2}, y, \bar{z} + \frac{3}{4}$                     |
| (11) $y, \bar{x}, \bar{z}$                                     | (12) $\bar{y} + \frac{1}{2}, x + \frac{1}{2}, \bar{z} + \frac{1}{2}$ |
| (13) $x + \frac{1}{2}, \bar{y} + \frac{1}{2}, z + \frac{1}{2}$ | (14) $\bar{x}, y, z$   |
| (15) $\bar{y} + \frac{1}{2}, \bar{x}, z + \frac{1}{4}$         | (16) $y, x + \frac{1}{2}, z + \frac{1}{4}$                           |

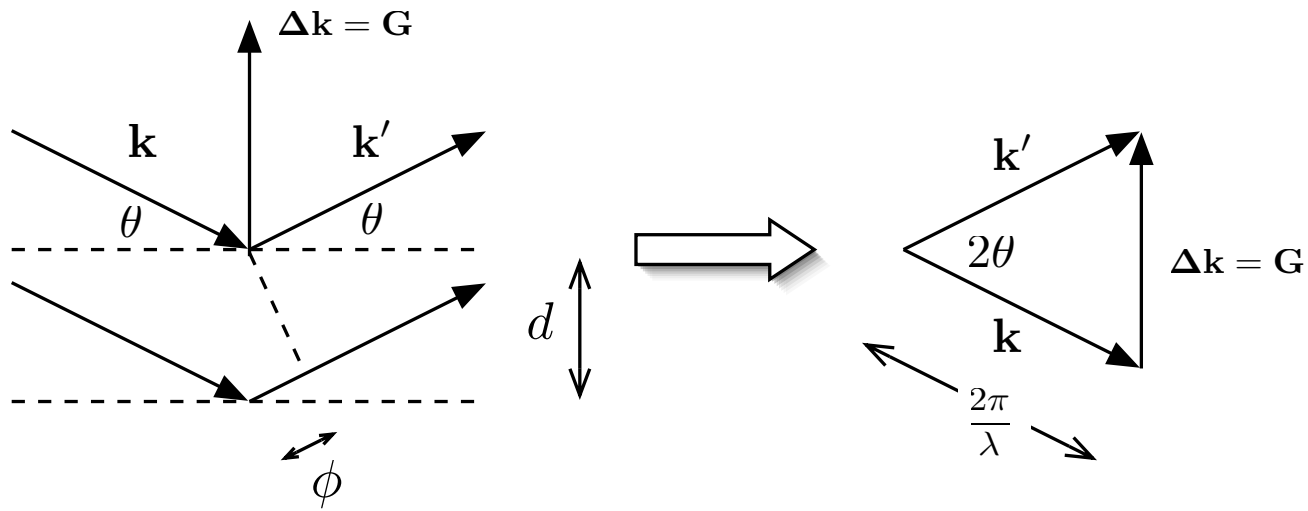
# Build Algorithm

```
Apply translation vectors to basis atom list
do(
  for each atom(
    for each transformation(
      Apply transform to atom
      Normalize the transformed location
      Create a copy of the atom at the
        transformed location
    )
  )
  remove duplicates from atom list
) while atom list is bigger than before, repeat do

remove atoms outside the unit cell
```

# XRD: Bragg Condition

$$\lambda = 2d \sin \theta \iff \Delta \mathbf{k} = \mathbf{G}$$



# Powder Diffraction

$$I(2\theta) = |F|^2 \cdot p \cdot \left( \frac{1 + \cos^2 2\theta}{\sin^2 \theta \cos \theta} \right)$$

Handling Multiplicities:

$$I(2\theta) = \sum |F|^2 \cdot \left( \frac{1 + \cos^2 2\theta}{\sin^2 \theta \cos \theta} \right)$$

# X-ray Diffraction Algorithm

```
for each  $h$  from  $-h_{\max}$  to  $h_{\max}$ :  
  for each  $k$  from  $-k_{\max}$  to  $k_{\max}$ :  
    for each  $l$  from  $-l_{\max}$  to  $l_{\max}$ :  
      Gmag =  $|h\mathbf{b}_1 + k\mathbf{b}_2 + l\mathbf{b}_3|^2$   
      if  $\text{Gmag} \cdot \lambda / 4\pi < 1$  :  
        add  $[h \ k \ l]$  to list of allowed  $\mathbf{G}$   
  
for each allowed  $\mathbf{G}_i$  :  
  for each basis atom $_j$ :  
     $\mathbf{R}_j$  = Atom $_j$ 's Centroid  
     $s = |\mathbf{G}_i| / 4\pi$   
     $F_i = F_i + f_j(s) e^{\mathbf{R}_j \cdot \mathbf{G}_i}$   
save  $F_i$ 
```

## Convert to $I(2\theta)$

for each  $G_i$  :

$$\theta_i = \sin^{-1}\left(\frac{\lambda |G_i|}{4\pi}\right)$$

$$I(2\theta_i) = I(2\theta_i) + |F_i|^2$$

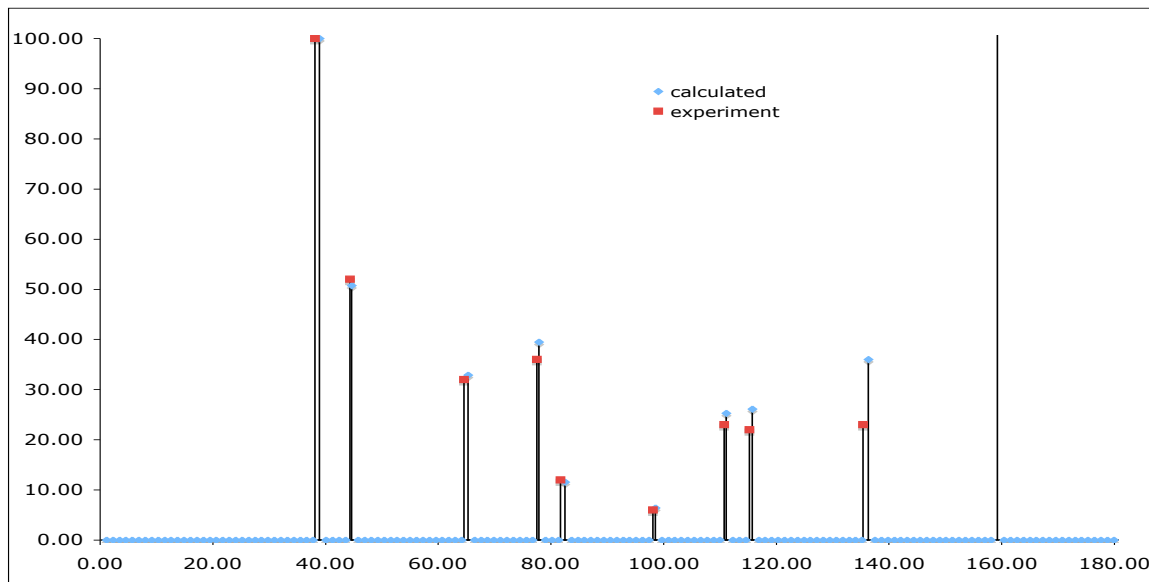
for each  $\theta_n$  :

$$I(2\theta_n) = I(2\theta_n) * LP(\theta_n)$$



# X-Ray diffraction comparison

## FCC Gold



Experimental data from Swanson and Tager 1953

# Classroom Demos

- Redefining the Unit Cell:  
Conventional BCC vs. Primitive BCC
- $\text{Cu}_3\text{Au}$  Order-Disorder Transformation:  
Simple Cubic to Face-Centered Cubic
- Martensitic Transformation:  
Cubic to Tetragonal





# Conclusions

- Build and Visualize Crystal Structure
- Simulate Powder Pattern
- All in an easy to use package

## Future Directions

- Classroom Use
- Availability online, Open Source
- Expanded diffraction capability, customizability

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<http://pruffle.mit.edu/atomiccontrol>

